

Problem 1.31

[Difficulty: 3]

1.31 In Section 1.6 we learned that the Manning equation computes the flow speed V (m/s) in a canal made from unfinished concrete, given the hydraulic radius R_h (m), the channel slope S_0 , and a Manning resistance coefficient constant value $n \approx 0.014$. For a canal with $R_h = 7.5$ m and a slope of $1/10$, find the flow speed. Compare this result with that obtained using the same n value, but with R_h first converted to ft, with the answer assumed to be in ft/s. Finally, find the value of n if we wish to *correctly* use the equation for BG units (and compute V to check!).

Given: Information on canal geometry.

Find: Flow speed using the Manning equation, correctly and incorrectly!

Solution: Use Table G.2 and other sources (e.g., Google) as needed.

The Manning equation is
$$V = \frac{R_h^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}}{n}$$
 which assumes R_h in meters and V in m/s.

The given data is $R_h = 7.5 \text{ m}$ $S_0 = \frac{1}{10}$ $n = 0.014$

Hence
$$V = \frac{7.5^{\frac{2}{3}} \cdot \left(\frac{1}{10}\right)^{\frac{1}{2}}}{0.014} \quad V = 86.5 \frac{\text{m}}{\text{s}}$$
 (Note that we don't cancel units; we just write m/s next to the answer! Note also this is a very high speed due to the extreme slope S_0 .)

Using the equation incorrectly: $R_h = 7.5 \text{ m} \times \frac{1 \cdot \text{in}}{0.0254 \cdot \text{m}} \times \frac{1 \cdot \text{ft}}{12 \cdot \text{in}} \quad R_h = 24.6 \text{ ft}$

Hence
$$V = \frac{24.6^{\frac{2}{3}} \cdot \left(\frac{1}{10}\right)^{\frac{1}{2}}}{0.014} \quad V = 191 \frac{\text{ft}}{\text{s}}$$
 (Note that we again don't cancel units; we just write ft/s next to the answer!)

This incorrect use does not provide the correct answer
$$V = 191 \frac{\text{ft}}{\text{s}} \times \frac{12 \cdot \text{in}}{1 \cdot \text{ft}} \times \frac{0.0254 \cdot \text{m}}{1 \cdot \text{in}} \quad V = 58.2 \frac{\text{m}}{\text{s}} \quad \text{which is wrong!}$$

This demonstrates that for this "engineering" equation we must be careful in its use!

To generate a Manning equation valid for R_h in ft and V in ft/s, we need to do the following:

$$V\left(\frac{\text{ft}}{\text{s}}\right) = V\left(\frac{\text{m}}{\text{s}}\right) \times \frac{1 \cdot \text{in}}{0.0254 \cdot \text{m}} \times \frac{1 \cdot \text{ft}}{12 \cdot \text{in}} = \frac{R_h^{(m)\frac{2}{3}} \cdot S_0^{\frac{1}{2}}}{n} \times \left(\frac{1 \cdot \text{in}}{0.0254 \cdot \text{m}} \times \frac{1 \cdot \text{ft}}{12 \cdot \text{in}}\right)$$

$$V\left(\frac{\text{ft}}{\text{s}}\right) = \frac{R_h(\text{ft})^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}}{n} \times \left(\frac{1 \cdot \text{in}}{0.0254 \cdot \text{m}} \times \frac{1 \cdot \text{ft}}{12 \cdot \text{in}}\right)^{-\frac{2}{3}} \times \left(\frac{1 \cdot \text{in}}{0.0254 \cdot \text{m}} \times \frac{1 \cdot \text{ft}}{12 \cdot \text{in}}\right) = \frac{R_h(\text{ft})^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}}{n} \times \left(\frac{1 \cdot \text{in}}{0.0254 \cdot \text{m}} \times \frac{1 \cdot \text{ft}}{12 \cdot \text{in}}\right)^{\frac{1}{3}}$$

In using this equation, we ignore the units and just evaluate the conversion factor

$$\left(\frac{1}{0.0254} \cdot \frac{1}{12}\right)^{\frac{1}{3}} = 1.49$$

Hence
$$V\left(\frac{\text{ft}}{\text{s}}\right) = \frac{1.49 \cdot R_h(\text{ft})^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}}{n}$$

Handbooks sometimes provide this form of the Manning equation for direct use with BG units. In our case we are asked to instead define a new value for n:

$$n_{\text{BG}} = \frac{n}{1.49} \qquad n_{\text{BG}} = 0.0094 \qquad \text{where} \qquad V\left(\frac{\text{ft}}{\text{s}}\right) = \frac{R_h(\text{ft})^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}}{n_{\text{BG}}}$$

Using this equation with $R_h = 24.6 \text{ ft}$:
$$V = \frac{24.6^{\frac{2}{3}} \cdot \left(\frac{1}{10}\right)^{\frac{1}{2}}}{0.0094} \qquad V = 284 \frac{\text{ft}}{\text{s}}$$

Converting to m/s
$$V = 284 \cdot \frac{\text{ft}}{\text{s}} \times \frac{12 \cdot \text{in}}{1 \cdot \text{ft}} \times \frac{0.0254 \cdot \text{m}}{1 \cdot \text{in}} \qquad V = 86.6 \frac{\text{m}}{\text{s}} \qquad \text{which is the correct answer!}$$